Traffic Engineering (CIVIL-349)
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Exercise 6 - Solution
Merging & Ramp Metering

Fall 2025

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Problem 1

a. The capacity envelope for the merging of the two highways is shown in figure 1, in blue color, while the priority line is shown in red.

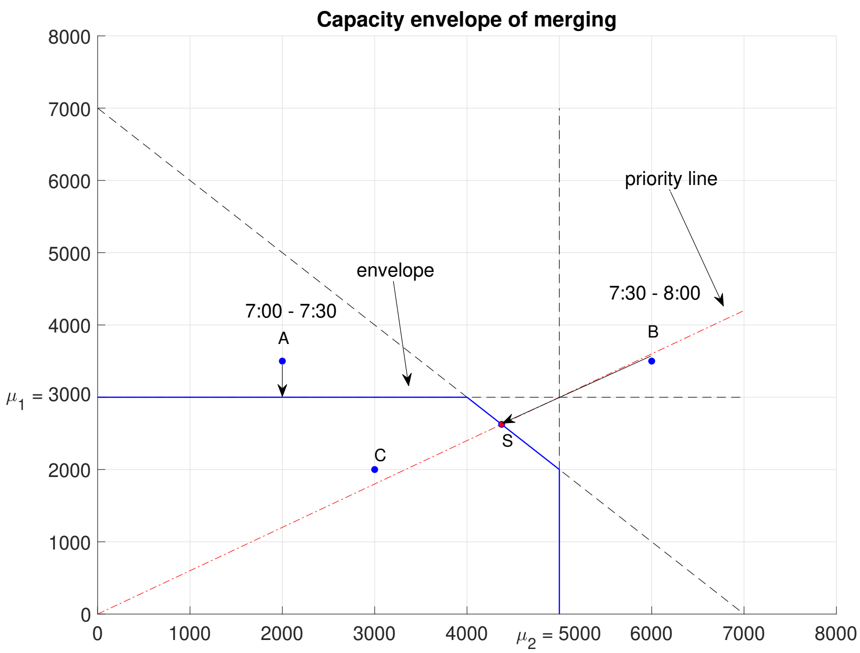


Figure 1: Capacity envelope (blue) and priority line (dashed red) for the merging of the two highways.

When both approaches are queued, the service point (proportional to their service capacities) is point S, the intersection between the capacity envelope and the priority line. The coordinates of point S can be found by splitting the bottleneck capacity proportionally to the capacities of the approaches, i.e. $q_1 = \frac{3000}{3000+5000} 7000 = 2625$ veh/h and $q_2 = \frac{5000}{3000+5000} * 7000 = 4375$ veh/h. [1]

[1] The service rates can equally be found by solving the system of the equations $q_1 = 7000 - q_2$ (bottleneck capacity line) and $q_1 = \frac{3000}{5000} q_2$ (priority line).

b. Before 7:00 am, there is no congestion and none of the approaches is queued. From 7:00 am to 7:30 am, only ramp 1 will be congested and a queue will start growing there, because the demand 3500 veh/h is higher that the capacity $\mu_1 = 3000$ veh/h (see demand point A in figure 1).

From 7:30 am to 8:00 am, both ramps will be congested (demand point B), so the two ramps will serve at priority rates (point S), with the rates $q_1 = 2625$ veh/h and $q_2 = 4375$ veh/h, which we specified in question a.

From 8:00 am and for the rest of the morning, the demand decreases in both approaches, but the service rate remains the same until the first queue dissolves. In order to observe the formation of queues, we draw the queueing (Input/Output) diagrams for each approach:

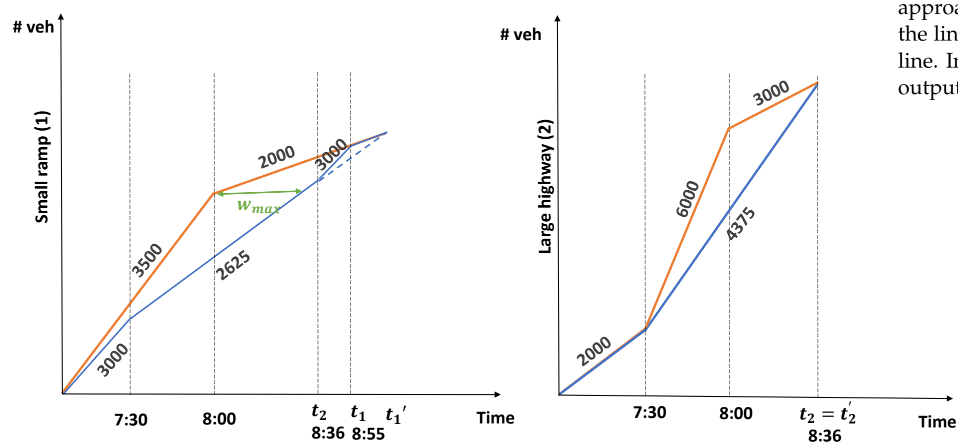


Figure 2: Schematic Input-Output diagrams at the entrance of the two approaches (not to scale). The slopes of the lines are written above or below the line. Input line is in orange color while output is in blue.

We denote t_1' and t_2' the times it will *theoretically* take for queues to dissipate on ramp 1 and 2, respectively. At first, we don't consider that once one of the two ramps is free of congestion (queue disappears), the service rates for the approach still congested will return to the maximum value, μ_1 or μ_2. We call t_1 and t_2 the *actual time* it will take for queues to dissipate on ramp 1 and 2, respectively.

In order to find t_1', the time that the queue in approach 1 will disappear assuming that approach 2 remains congested, we can say that at $t = t_1'$, the total cumulative input will be equal to the total cumulative output. This means that if 7:00 am is $t = 0$:

$$\text{Input: } 3500 \times 1 + 2000 \times (t_1' - 1) \text{ veh}$$
$$\text{Output: } 3000 \times 0.5 + 2625 \times (t_1' - 0.5) \text{ veh}$$

By solving the equation Input = Output for t_1', we find that $t_1' = 2.1$ h. Similarly, under the same assumption for ramp 2, we have:

$$\text{Input: } 2000 \times 0.5 + 6000 \times 0.5 + 3000 \times (t'_2 - 1) \text{ veh}$$
$$\text{Output: } 2000 \times 0.5 + 4375 \times (t'_2 - 0.5) \text{ veh}$$

By solving the equation Input = Output for t'_2, we find that $t'_2 = 1.6$ h. This means that *the queue will first disappear on the large highway 2*, at time 7:00 am + 1.6h = 8:36 am, before the small highway 1. However, when highway 2 is free of congestion, and since there is still a queue in highway 1, the service rate of 1 will become $\mu_1 = 3000$. Hence, by updating the I/O diagram of highway 1, we include the new maximum service rate μ_1 from 8:36 am ($= t_2$), until the queue disappears at t_1. To find t_1 we can write:

$$\text{Input: } 3500 \times 1 + 2000(t_1 - 1) \text{ veh}$$
$$\text{Output: } 3000(0.5) + 2625(1.6 - 0.5) + 3000(t_1 - 1.6) \text{ veh}$$

By solving the equation Input = Output for t_1, we find $t_1 = 1.91$ h, which means at 8:55 am.

c. It is clear that the unlucky driver is the one who arrived at ramp 1 at 8:00 am, because since this moment the queue starts decreasing. A rapid check proof that this driver will leave the ramp before t_2, that is the time when the capacity on ramp 1 will increase again at 3000 veh/h is the following:

$$3000 \times 0.5 + 2625 \times (1.6 - 0.5) = 4387.5 > 3500 \times 1$$

The waiting time is the difference between the input line and the output line on the same level (3500). We can easily compute the time interval x between 7:30 am and the exit time of the unlucky driver by the following equation:

$$3000 \times 0.5 + 2625 \times x = 3500 \Rightarrow x \simeq 0.76$$

Then, considering that the driver arrived at 8:00 am, we have that the max waiting time is $7.5 + 0.76 - 8 = 0.26$ hours. So, the unlucky driver experienced a delay of 0.26 h $= 15$ min 36 sec.

d. The maximum queue length in the highway 2 can be found, based on the I/O diagram, as follows:

Until 7:30 am, there is no queue in highway 2. From 7:30 to 8:00 am, the arriving demand is 6000 veh/h while the service rate is 4375 veh/h. At 8:00 am, the demand drops below the capacity; therefore, at this moment, the queue starts decreasing, so it is maximum at 8:00 am. We can find that the number of vehicles in queue at 8:00 am is $(6000 - 4375) \times 0.5 = 812.5$ veh.

e. If we assume that the user arrives at the merge at 8:30 am, we need to calculate her waiting time for both cases.

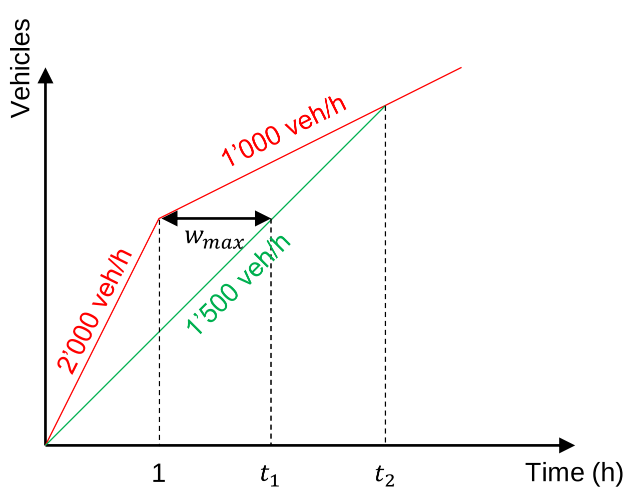
For highway 2, we know that the queue dissolves with a rate of $3000 - 4375 = -1375$ veh/h and it disappears at 8:36 am. This means that at 8.30 am, there are $6/60 \times 1375 = 137.5$ veh in the queue. Since the service rate is 4375 veh/h, the necessary time for this queue to disappear is $137.5/4375 = 0.0314$ h $= 1.88$ min.

For highway 1 we know that at 8:30 am, $3500 + 2000 \times 0.5 = 4500$ veh have joined the queue (input) and $3000 \times 0.5 + 2625 \times 1 = 4125$ veh have left (output), so there are $4500 - 4125 = 375$ veh currently in the queue. For 6 minutes, until 8:36 am, the service rate is 2625 veh/h, which means that during this time only $6/60 \times 2625 = 262.5$ veh can be served, which is smaller than the existing queue of 375 veh. This means that the driver arriving at 8:30 am in highway 1, will wait more that 6 mins to pass, which is already longer than the 1.88 mins that we found for highway 2.

Therefore, if both paths leading to the merge have equal travel times, the user should choose the path leading to the large highway 2, where she will experience shorter delay.

Problem 2

a. Note that since no vehicle will change their route, all vehicles that arrive at the ramp will pass through it. Hence, the Input-Output diagram is as follows:



Therefore, $2000 \times 1 = \mu_1 \times t_1 \Leftrightarrow t_1 = 4/3h$. As $w_{max} = t_1 - 1$, $w_{max} = 0.33h = 20min$.

The queue dissipates at t_2 hours.

$$1500t_2 = 2000 \times 1 + 1000 \times (t_2 - 1) \tag{1}$$

i.e. $t_2 = 2$ hours.

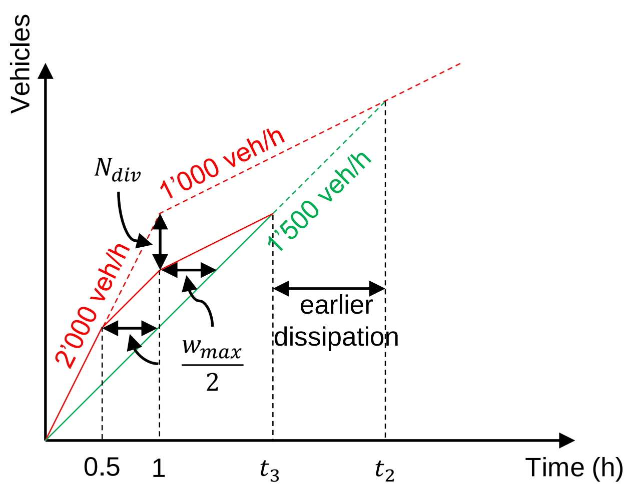
b. This question requires solving the dynamics of queues until the queue disappears in both scenarios. Let w_1 denote the waiting time on the ramp. Wardrop's principle implies that users choose the alternative route if and only if

$$w_1 >= \frac{w_{max}}{2} \tag{2}$$

i.e. $w_1 >= 10$min.

Note that since there is no capacity constraint on the alternative route, the delay at ramp cannot exceed 10min. Indeed, as soon as it is slightly greater than 10min, all users would choose the alternative route and the queue would return to 10min. Hence, whenever the arrival flow is greater than the capacity of the bottleneck for some period, the queue builds up until it reaches the value of 10min and then remains in this state, which is an equilibrium.

Let t_3 denote the time at which the waiting time reaches this equilibrium value.



The input-output diagram for the ramp allows us to find t_3 easily. The number of vehicles that have arrived at the ramp at 1h is given by $2000 \times 0.5 + 1500 \times (1 - 0.5)$. Hence the number of vehicles that diverted (chose to travel through the alternative route) is

$$N_{div} = (2000 - 1500) \times (1 - 0.5) \tag{3}$$

So $N_{div} = 250$ vehicles.

Because of the change in the route of these vehicles, the queue will dissipate earlier than t_2; this will instead happen at time t_3.

$$1500t_3 = 2000 \times 0.5 + 1500 \times 0.5 + 1000 \times (t_3 - 1) \Leftrightarrow t_3 = 1.5 \text{ hours} \tag{4}$$

i.e. the queue dissipates 0.5 hours earlier.